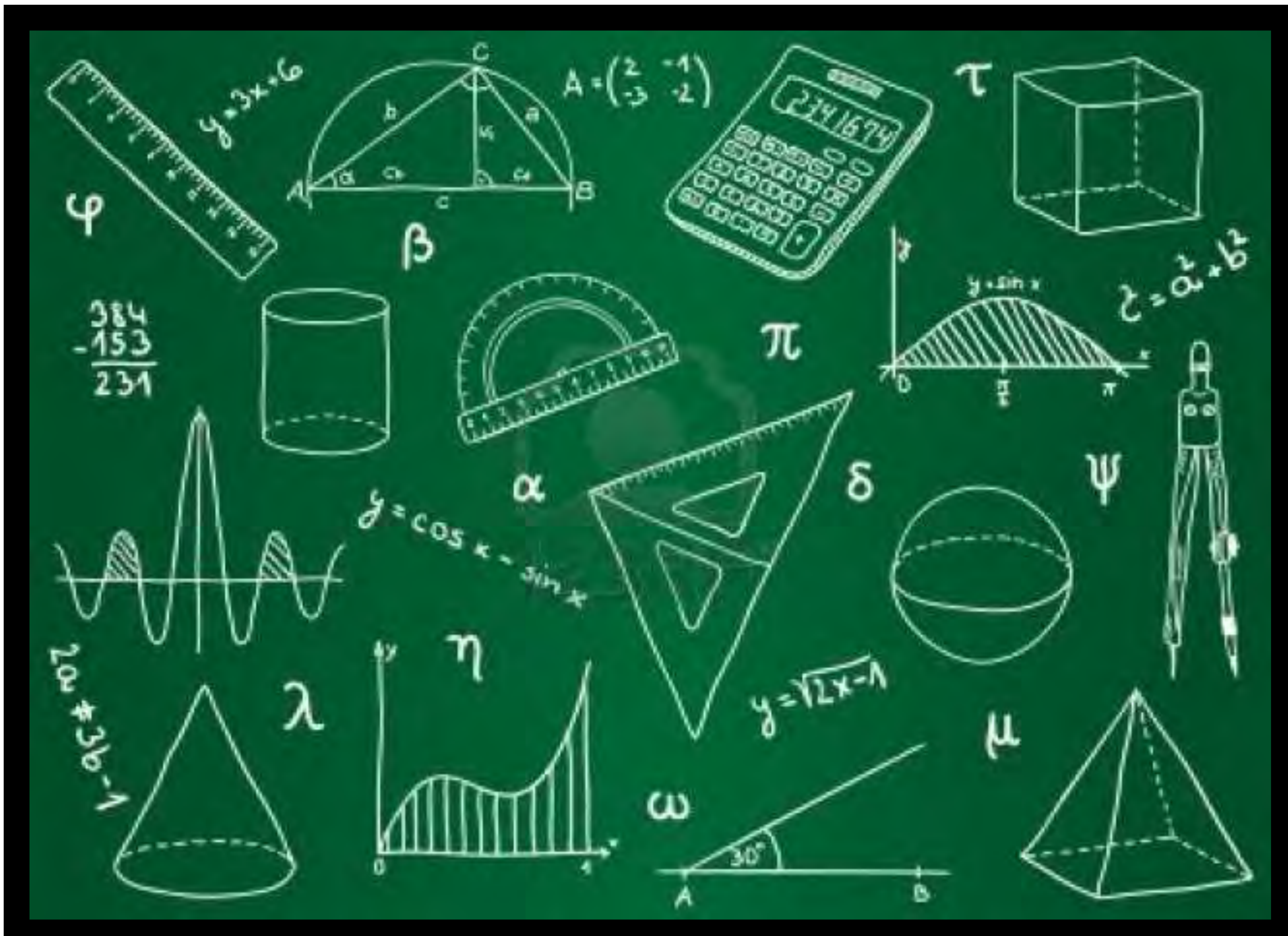
Oklahoma Journal of School Mathematics



Official Publication of the
Oklahoma Council of Teachers of Mathematics

Fall 2013, Volume 5, Issue 2

Journal Editors

Juliana Utley
School of Teaching and Curriculum Leadership
Oklahoma State University
233 Willard Hall
Stillwater, Oklahoma 74078
juliana.ultey@okstate.edu
405.744.8111

Stacy Reeder
Instructional Leadership and Academic Curr.
University of Oklahoma
820 Van Vleet Oval, ECH 114
Norman, Oklahoma 73019
reeder@ou.edu
405.325.1498

OCTM Officers

Oklahoma Journal of School Mathematics

Fall 2013 Volume 5, Issue 2

Letter from the Editors

The Common Core Standards for Mathematical Practices describe mathematical habits of mind that teachers should help students develop. One of these practices points to the need for students to "construct viable arguments and critique the reasoning of others" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). Frequently teachers interpret this practice with a narrow focus, believing that having students write their explanations is the best way to address and develop this practice. While having our students write in mathematics class is incredibly important, it is not the only way to develop their abilities to reason mathematically. In fact, teachers who engage their students in mathematical discourse on a regular basis may well be providing their students with rich opportunities to reason mathematically and to defend and understand the reasoning of others.

When students are required to share their thinking or a strategy for problem solving it pushes them to clarify and solidify their mathematical understanding. Chapin, O'Conner, and Anderson (2013) suggest that there are "five reasons talk is critical to teaching and learning" (p. xv): (1) talk can reveal understanding and misunderstanding, (2) talk supports robust learning by boosting memory, (3) talk supports deeper reasoning, (4) talk supports language development, and (5) talk supports development of social skills. The importance of mathematical discourse and student discussion in mathematics classes is not a new idea. We simply need to embrace it and provide our students the necessary environment to explain their thinking – first by talking and then in writing!

Juliana Utley Stacy Reeder

For a review of Chapin, O'Conner, and Anderson's (2013) book *Classroom Discussions in Math* see page 26. Citation: Chapin, S. H., O'Conner, C., & Anderson, N. C. (2013). *Classroom Discussions in Math: A Teacher's Guide to Using Talk Moves to Support the Common Core and More*. Sausalito, CA: Math Solutions.

Teaching Integer Arithmetic without Rules: An Embodied Approach

By Thomas J. Faulkenberry and Eileen D. Faulkenberry

When teaching mathematics courses for prospective mathematics teachers, we spend a great deal of time trying to help these future teachers develop a broad conceptual understanding about the most basic concepts of arithmetic. In accord with the expectations of NCTM's *Principles and Standards for School Mathematics* (2000), we use questions like "What does the addition sign really *mean*?" and "How many different ways can you assign meaning to the concept of division?" to get our preservice teachers to think more deeply about the mathematics they will be teaching. The class discussion generated from these questions begins to acquaint the future teachers with the ideas of Cognitively Guided Instruction (Carpenter et al., 1999).

As an example, consider the problem 7 - 3. We asked a class of pre-service mathematics teachers to model this arithmetic problem with a word problem. Out of 32 students, all constructed a problem that involved starting with seven objects and removing three of them to see how many were remaining. Typical examples included, "There are seven birds in a yard and three fly away. How many are left?" In the language of Carpenter et al. (1999), this model is an example of the problem type *Separate (result unknown).* We tried to generate some more examples in class, but everyone agreed that any model for 7 - 3 would look similar to ones they had already done.

Of course, there are other models for 7 - 3. Consider the following example:

"Jim has 3 golf balls. Sidney gives him some more golf balls. Now Jim has 7 golf balls. How many did Sidney give to Jim?"

Carpenter et al. (1999) identified this problem type as *Join (change unknown).* It is a perfectly good model for 7 - 3. However, the question remains: *Why did none of our students think of examples that would be classified as a join problem?*

The same issues regarding the formation of meaning in mathematics also come up when we work with in-service teachers. During a recent dialogue with a group of in-service teachers at a professional development workshop, the topic of integer arithmetic came up. Specifically, one of the teachers asked, "How do we teach integer arithmetic so that it *makes sense* to the students without them having to memorize a bunch of rules?" We proposed using the *Balloons and Sandbags* activity (Herzog, 2008). This activity is easy to understand, requires few classroom materials, and it is based on a principle from cognitive psychology known as *embodied mathematics.*

Embodied Mathematics

Embodied mathematics is a natural outgrowth of scientific work in embodied cognition (Pecher & Zwaan, 2005; Wilson, 2002). It was brought to a wider audience with the work of G. Lakoff and R. Nunez (2000) in their book, *Where Mathematics Comes From.* Lakoff and Nunez make a strong case that all mathematical thought, even the most advanced mathematics, is a result of humans using *conceptual metaphors* to conceptualize abstract thought in terms of embodied, sensory-motor experiences. In other words, we think about abstract mathematical concepts in terms of much simpler, body-based experiences. To understand how this works, let's first consider a non-mathematical example of a conceptual metaphor.

A conceptual metaphor (Lakoff & Johnson, 1980) is not exactly the same thing as a figure of speech (a literary metaphor). Rather, a conceptual metaphor is a way of thinking about something abstract in terms of something else with which we have prior sensory-motor experience. For example, consider the metaphor AFFECTION = WARMTH. As the reader may recognize, this metaphor is at the root of many conventional American English expressions, such as:

- "She *warmed* up to him"
- "We haven't *broken the ice* yet"

Notice that in these linguistic examples, there is a strong correspondence between the structure of the abstract idea *affection* and the embodied idea of *warmth*. Affection is conceptualized as warmth, whereas disaffection is conceptualized as the opposite of warmth, or cold.

How do these ideas apply to mathematical thought? Lakoff and Nunez (2000) present many conceptual metaphors that they believe form the basis for our mathematical ideas. For the purposes of this article, we will only focus on one metaphor that underlies simple arithmetic: *arithmetic = object collection*. With this metaphor, the ideas of simple arithmetic (order, addition, subtraction) are thought of in terms of the embodied experience of collecting objects. Specifically, the structured metaphorical correspondence is listed in Table 1.

OBJECT COLLECTION	ARITHMETIC
Collections of objects the same size	Numbers
The size of the collection	The size of the number
Bigger	Greater
Smaller	Less
The smallest collection	The unit (One)
Putting collections together	Addition
Taking a smaller collection from a larger collection	Subtraction

Table 1: The conceptual metaphor "ARITHMETIC = OBJECT COLLECTION"

Notice that this seems to be precisely what is guiding the *Join/Separate* principles in Carpenter et al. (1999). In the language of embodied mathematics, addition is simply a metaphorical extension of the embodied experience of putting two collections together (joining), and subtraction is a metaphorical extension of the embodied experience of taking a smaller collection away from a larger collection (separating).

At this point, we should have some insight to our original problem: why did none of our students think of the "addition with a missing addend" model for 7 - 3? Using the ideas of embodied mathematics, we posit the following explanation. Because the students are seeing the operation as subtraction, and since subtraction is conceptualized (via the ARITHMETIC = OBJECT COLLECTION metaphor) as a process of *taking away*, each of the students' models involved taking away. There is no reason, cognitively, for a subtraction problem such as 7 - 3 to have been modeled as $3 + N = 7$, since the latter is perceptually an *addition* problem. Mathematically, they are the same, but *cognitively*, they are completely different.

Based on this basic arithmetic metaphor, does it follow that integer arithmetic is structured cognitively as object collection? Integer arithmetic is often difficult for students of all ages to master because it seems to be just a collection of rules; that is, it lacks an embodied foundation. For example, to calculate 3 - (-5), one simply changes the two negatives to a positive and adds 3 and 5 to get 8. This, of course, is one of the easier rules; the rest can be even more difficult or confusing to students. So, when teachers approach us about how to teach integer arithmetic for understanding using embodied mathematics, we often look to the *Balloons and Sandbags* activity (Herzog, 2008).

Embodied Integer Arithmetic with Balloons and Sandbags

Balloons and Sandbags is a conceptual model of integer arithmetic where each balloon represents a positive quantity and each sandbag represents a negative quantity. The idea stems from the physical interpretation that a balloon lifts upward and a sandbag would fall to the ground. The basic idea is that each balloon exerts an equal and opposite force to each sandbag. So, for example, an addition problem such as $3 + (-2)$ could be modeled as starting with three balloons (an upward lift of 3 units) and adding two sandbags (a downward pull of 2 units), resulting in a net upward lift of 1 unit (see Figure 1). Hence, $3 + (-2) = 1$. Notice that with this representation, the operation of addition is still conceptualized as "putting two collections together." Contrast this with the "rule" which states "a plus and a minus together equals a minus, so it's three minus two." In this case, a student is forced to give up his or her embodied conception of addition as "joining" in favor of a rule with no reason. At best, this perplexes the student; at worst, it reinforces the negative stereotype that mathematics is just a bunch of rules.

Any integer addition or subtraction problem can be modeled with the Balloons and Sandbags activity. To guide our discussion, we consider two main classes of problems: problems involving addition (joining) and problems involving subtraction (separating).

Addition Problems

For example, consider the problem $-4 + 7$. This problem would be modeled by starting with 4 sandbags (to represent downward pull of 4 units embodied in the integer -4). The action that occurs is adding 7 balloons. Since every balloon exerts an equal and opposite force on every sandbag, the 4 sandbags pair with 4 of the newly added balloons so that each pair gives a net lift of 0 units (see Figure 2). All that remains are 3 balloons, which each have positive lift. Hence, the answer is $-4 + 7 = 3$.

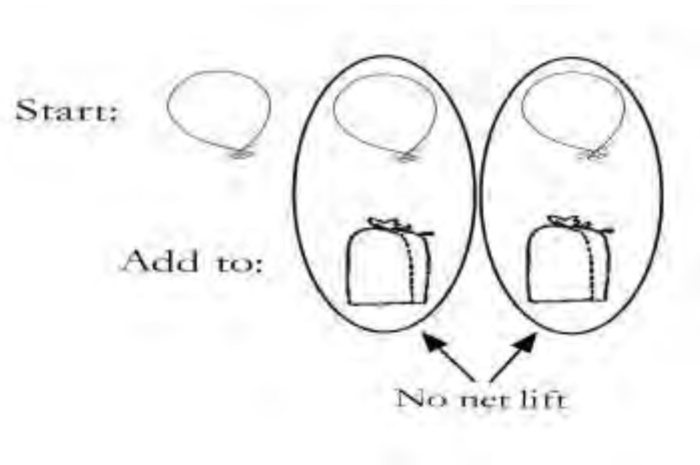


Figure 1. Balloons and sandbags representation of 3 + (-2).

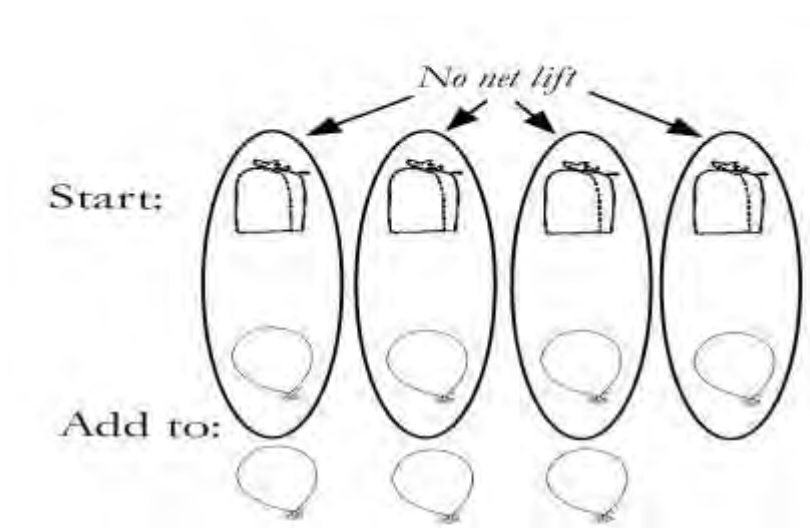


Figure 2. Balloons and sandbags representation of -4 + 7

As another example, consider –3 + (-4). Again, this problem would start with 3 sandbags, to which we would add 4 more sandbags (see Figure 3). This gives a total of 7 sandbags, resulting in a net lift of –7. Hence, the answer is –3 + (-4) = -7.

Subtraction Problems

For example, consider the problem 3 - (-5). Note that even though the popular strategy for solving this problem involves changing this to 3 + 5, this is technically a subtraction problem. Using the Balloons and Sandbags model, we may model this by starting with three balloons. The problem is asking us to remove five sandbags from this collection. This is impossible to do since we are only starting with three balloons. This requires the addition of what we technically term "zero pairs;" that is, balloon/sandbag pairs with a net lift of 0 units. Through class discussion, we discuss what different configurations of balloons would result in the same net lift as three balloons would; for example, starting with 3 balloons, no number of zero pairs that we add to this collection will change the net lift of 3 units provided by these 3 balloons. Specifically, 3 balloons with 5 balloon/sandbag pairs would still exert a net lift of 3 balloons. However, this configuration does indeed have 5 sandbags to remove; if we remove them, we are left with 8 balloons (see Figure 4). Hence, 3 - (-5) = 8. Once again, the “taking away” conception of subtraction is preserved, as opposed to the rule that says that this subtraction problem is really an addition problem. Note, however, how the solution of this problem through the Balloons and Sandbags activity does indeed give some insight as to why this rule works in general, giving the student a chance to discover the rule on his/her own. Since the student added five balloon/sandbag pairs to the configuration in order to have five sandbags to remove, the idea of addition has been introduced to the situation. Also, since the sandbags were removed but the balloons remained, the student actually added the opposite (the balloons) of what he wanted to take away (the sandbags), thus leading to the idea of subtraction being equivalent to “adding the opposite.”

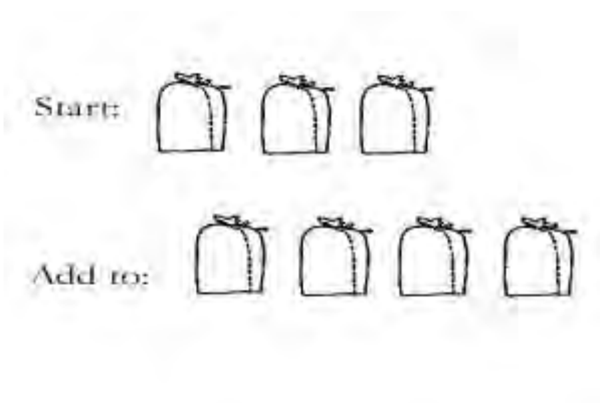


Figure 3. Balloons and sandbags representation of -3 + (-4)

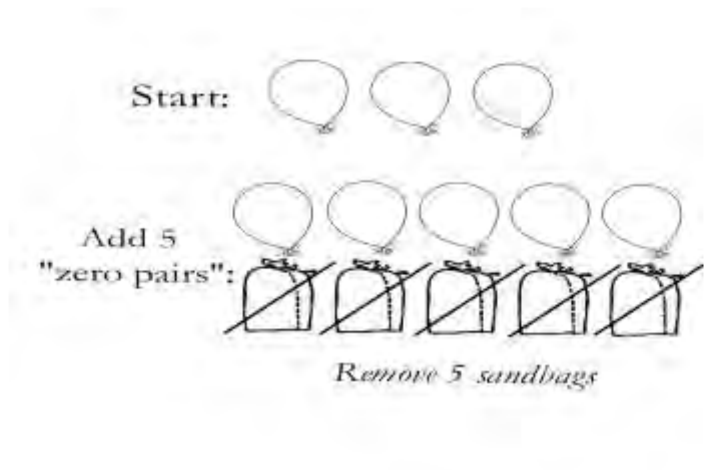


Figure 4. Balloons and sandbags representation of 3 - (-5)

As another example illustrating the technique of adding zero pairs, consider the example –3 – 4. The more mathematically sophisticated student will notice that this problem has the same answer as the problem –3 + (-4) that we solved above. However, let us model the problem as it is presented: as a subtraction problem. The problem states that we start with 3 sandbags, and the intended action is to remove 4 balloons. Since we have no balloons, we need to introduce them without changing the net lift of the system; again, we can accomplish this by adding zero pairs. To be able to remove 4 balloons, it is sufficient to add 4 balloon/sandbag pairs. After we remove the 4 balloons, one can see that we are left with 7 sandbags (see Figure 5). Hence, the solution to the problem is –3 –4 = -7.

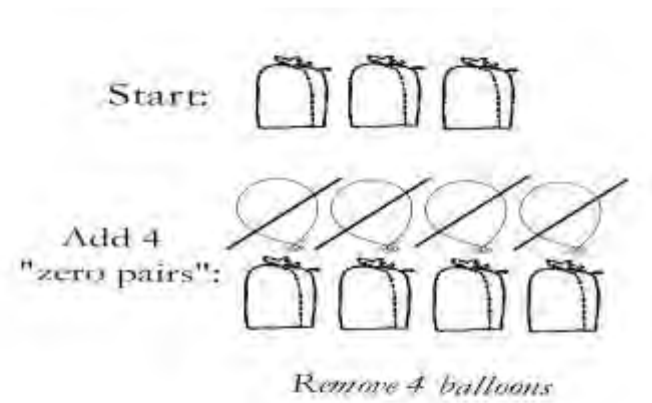


Figure 5. Balloons and sandbags representation of -3 - 4

Classroom Issues

The Balloons and Sandbags model is easy to use in the classroom. The only materials needed are something to represent balloons and something to represent sandbags. For children just beginning to investigate integer arithmetic, teachers may decide to provide pictures of balloons and pictures of sandbags that are copied onto cardstock and cut out with scissors. Children may then model their problems through manipulating these concrete materials in the manner discussed above. Children who have had some exposure to this activity with concrete materials will begin to solve problems by simply drawing their own balloons and sandbags on paper and recording their actions (see Figures 6 and 7). In Figure 6, a sixth-grade student is solving the problem -2 + 6. She begins with two sandbags to represent the -2. Then she adds 6 balloons. Next, she circles pairs of balloons and sandbags to create "zero pairs." Once she has made all the zero pairs she can, she knows that the remaining pieces represent the answer. She has four balloons left so writes her answer as 4.

Figure 7 shows an eighth-grade student solving a subtraction problem, 3-8. She begins with three balloons. She writes that she needs to take away 8 balloons so she must add balloon/sandbag pairs. Observing the student, we asked her how she knew how many balloon/sandbag pairs to add. She said she knew she needed 5 pairs because $3 + 5 = 8$. Once the student had 8 balloons to remove, she crossed the balloons out leaving five sandbags.

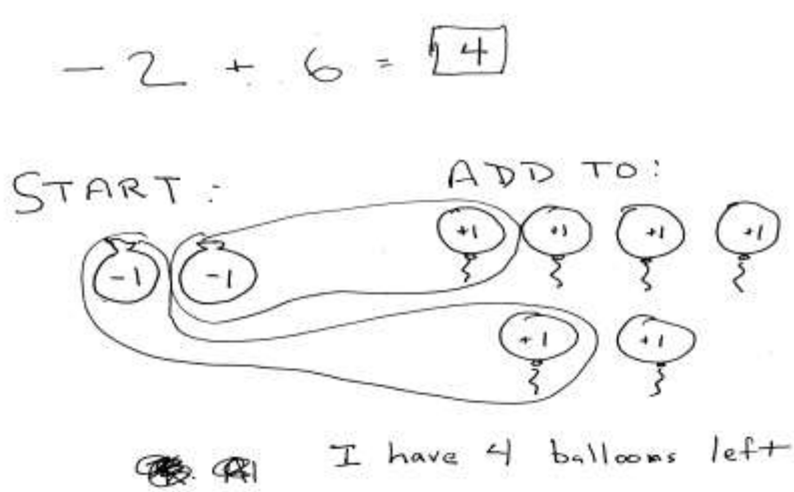


Figure 6. An example of a 6th grader's work on -2 + 6

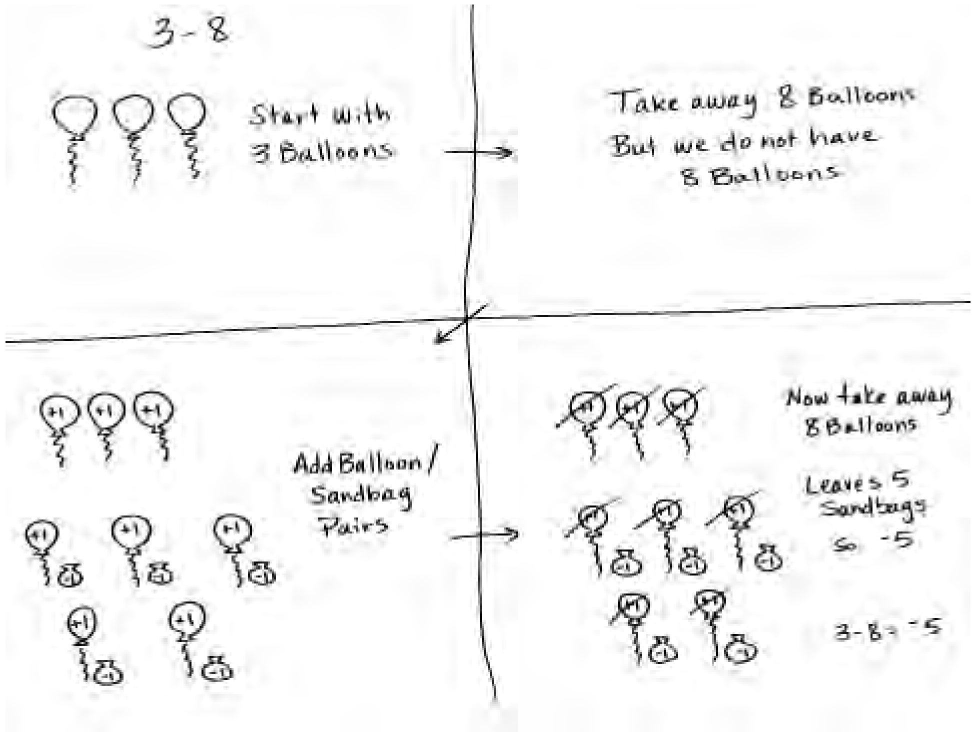


Figure 7. An example of an 8th grader's work on 3 - 8

Conclusion

These methods of teaching integer arithmetic are only a small sample, but they are indicative of a class of methods that appeals not only to good pedagogical sense, but also to current research in cognitive psychology. Some people may argue that the Balloons and Sandbags model is simply another set of rules for adding and subtracting integers. While there are certainly some "rules" for implementing the Balloons and Sandbags model correctly, these rules are based on the embodied notions of addition as "joining together" and subtraction as "taking away from." Contrast this with some of the other popular sets of rules for integers that involve switching signs according to certain patterns. These rules may often be efficient, but this efficiency comes at the expense of contradicting a student's embodied sense of arithmetic.

References

Carpenter, T. P., Fennema, E. Franke, M. L., Levi, L., Empson, S. B. (1999) *Children's Mathematics: Cognitively Guided Instruction.* Portsmouth, NH: Heinemann.

Herzog, D. A. (2008). *Teaching yourself VISUALLY algebra.* Hoboken, NJ: Wiley Publishing Inc.

Lakoff, G., & Johnson, M. (1980). *Metaphors we live by.* Chicago: University of Chicago Press.

Lakoff, G., & Nunez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being,* New York: Basic Books.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.

Pecher, D., & Zwaan, R. A. (2005). Introduction to grounding cognition: The role of perception and action in memory, language, and thinking. In D. Pecher & R. A. Zwaan (Eds.), *Grounding cognition: The role of perception and action in memory, language, and thinking.* (pp. 1-7). Cambridge, UK: Cambridge University Press.

Wilson, M. (2002). Six views of embodied cognition. *Psychonomic Bulletin & Review, 9,* 625-636.

Thomas J. Faulkenberry is an Assistant Professor of Psychology at Tarleton State University. Current research interests include numerical cognition.

Eileen D. Faulkenberry is an Associate Professor of Mathematics at Tarleton State University.

Toddler Engagement in Math through Play at a Child Development Center

By Jill M. Davis

Mathematics is an important part of young children's lives. They use mathematics to solve problems, order their universe, and make sense of their world (Geist, 2009). Evidence of mathematical thinking can be observed throughout the early childhood classroom as children place one cup in front of each chair at the dramatic play center, pour sand into different sized containers at the sensory table, and sort manipulatives into their containers during clean-up time. However, children construct mathematical ideas long before entry into school. In fact, it is during the first 5 years that children begin to learn the concepts that will support the formal math learned later in life (Hansen, 2005; National Council of Teachers of Mathematics, 2000). However, there is an extremely limited amount of research on mathematical knowledge of children under the age of 5, especially toddlers (Björklund, 2008; Reikerås, Løge, & Knivsberg, 2012; Sarama & Clements, 2009). The purpose of this research was to explore how toddlers, ages 1 to 3, engage in mathematics through play at a child development center.

Literature Review

Researchers support the idea that infants begin to develop an understanding of mathematical concepts beginning at birth (Clements, 2005; Greenberg, 2012; Ozdogan, 2011; Reikerås, et al., 2012). In fact, Geist (2001) proposed that children have a *mathematical acquisition device* that allows them to attain mathematical concepts without direct teaching. He analogizes this to Chomsky's theory of an innate language acquisition device. Just as children are born ready to learn language and build literacy concepts, they are also born ready to develop mathematical understandings. This mathematical acquisition device enables young children to construct the foundation necessary for later mathematical understanding, described as *emergent mathematical understanding.* He believed that the earliest understandings occur during the first few months of life, long before children are able to verbally express their ideas. They use actions to demonstrate what they are unable to articulate through language. These actions are often expressed through play.

Lee (2012) studied the mathematical play of young children between 13 months and three years of age. She analyzed videotapes of the children's actions and interactions and identified seven categories of mathematical play: space, number, measurement, shape, pattern, classification, and problem solving. Space included spatial understanding and explorations of space. Number included counting, cardinal knowledge, and the use of numbers as symbols. Measurement concepts included discussion of non-standard measurement, such as volume and weight. Naming, representing, and manipulating geometric shapes were included in the category of shape. Patterns included repetitions of actions and songs. Classification included sorting objects, data, and ideas. Problem solving included using strategies to solve problems both physically and cognitively. The first six categories were observed in individual episodes of play while problem solving was demonstrated in the other six categories. Lee's observations demonstrated that a variety of mathematics can be seen in a toddler classroom.

Research Methodology

The research question asked was *what kinds of math concepts do toddlers, ages one to three, engage in during play at a child development center?* This qualitative, single-case study took place in a toddler classroom at a child development center that also served as a laboratory school at a community college. It operated as a year-round, full-time childcare facility for faculty, staff, and members of the community. Purposeful sampling determined that the child-centered, play-based orientation of the classroom would make it appropriate for this study.

Participants in this study included the children and their teachers. There were 12 toddlers, 6 males and 6 females, ranging in ages from 19- to 31-months old. Ten children (83.3%) were Caucasian, one child (8.3%) was Native American, and one child (8.3%) was African American and Native American mixed. All of the children and their families spoke English as their native language. All of the parents in the class gave consent for their children to participate in the study. The two lead teachers in the classroom were both Caucasian females. Katie, age 23, worked 7:00 a.m. to 4:00 p.m. At the time of the study, she was working toward her Bachelor's degree in Early Childhood Education. As the former lead teacher in the younger toddler class, she looped up with 8 of the 12 toddlers in this study. Tonya, age 29, worked 10:00 a.m.to 6:00 p.m. She had completed a Master's degree in Psychology. She had several years of experience working with preschoolers but this was her first experience as the lead teacher in a toddler classroom. While there were several assistant teachers and student teachers present during data collection, the focus of the study was on the toddlers' primary caregivers.

I collected data from three primary sources: observations, interviews, and field notes. At various times of the day, I observed self-guided play. Although I initially positioned myself in areas likely to have the most opportunity for mathematical play such as the block or manipulative center, when those areas were vacant, I moved to other occupied areas. The play episodes were audio-recorded and field notes were written during eight 30 minute collection periods. I transcribed the tapes and used field notes to expand the context. I conducted semi-structured interviews with the two lead teachers, focusing on the kinds of math concepts that the toddlers engaged in during play. These interviews were also audio-recorded and, later, transcribed. Member checks were done with the two lead teachers to determine accuracy. Using a starter list of codes taken from the literature, I read over the transcriptions from the interviews and observations several times. These codes corresponded to Lee's (2012) seven categories of toddler math concepts during play, including numbers, space, measurement, shape, pattern, classification, and problem solving. I triangulated the data with entries from the field notebook.

Research Findings

There were three mathematical categories there were most often seen in the data: classification, number sense, and shape.

Classification. Classification, including early skills of matching and sorting, appeared in the data most often. Beatrice (26 months) matched shapes that the teacher taped on the ground.

> Beatrice notices a large, green circle that Tonya had just taped to the ground. She walks to the easel where there are many laminated shapes. She chooses a circle and lays it on the large, green circle on the floor. Beatrice repeats this several times, choosing a small circle from the easel and laying it on the circle on the floor. When there are no more circles, she uses squares and the large, purple square on the other side of the room.

Most of the toddlers only sorted by a single attribute, generally color. For example, Landon (26 months) placed each of the plastic, colored farm animals in the corresponding bowls. When he finished, he dumped out all of the animals and sorted by color again. Although this was typical for most toddlers, Katie described how Zander (27 months) demonstrated his mathematical understanding using the same manipulatives.

> He'll first group them by color and then he'll take them all out and group them by animal, not even looking at what the color is. So he might have a purple, blue, and green cow all grouped together, but they're all cows.

She compared this to some 4-year-olds who "were only able to group them one way where this child is 2 years old and he's grouping them in two different ways." Katie considered this ability to see more than one attribute in the same object to be atypical for Zander's age.

Number sense. The second most often appearing category was number sense. This included one-to-one correspondence, counting, and numerical understanding. Katie described how Beatrice demonstrated both one to one correspondence and counting while she played with construction materials and a jack in the box.

> We have bristle blocks on the ground and we have this pop-up toy. She [Beatrice] had it standing up, and she lined the bristle blocks on the ground as steps going up to the pop-up toy. She had an animal and she was going one by one on the bristle block, counting them: "One, two, three, four" until she got up to the door.

Both teachers differentiated between rote counting and understanding that numbers have meaning. Tonya shared an example of a child who had an understanding of number concepts.

> They just automatically say the next number without having to sit there with their fingers (counting) one, two, three. You know, without having to count to find out what happens next. That would be the one that I would say, 'Oh! He's really understanding the number concepts and numbers.' Just because they know how to count to 10 doesn't mean that they understand what they're doing… They could have memorized it because they've been counted with. You know, it's great that they can count, but does it mean that they actually understand they're counting objects? That those numbers mean something? Or is it just like singing their ABC's?

Katie supported this and stated that many of the toddlers are starting to understand that "the number represents something." According to her, the goal is for the toddlers to move beyond rote memorization.

Many of the older toddlers were observed counting. Nadine (31 months) lined up the toy animals and counted, "One, two, three, four, five, six". However, not all of the toddlers counted with understanding. Some of the toddlers used number words but did not necessarily use them in the correct order, especially when counting above three. McKenzie (25 months) played with a group of toy people. She counted them, pointing at some people twice and skipping others, "One, two, three, five, seven, nine!" Katie considered this and stated, "She is starting to count but doesn't count in the correct order. So she is *starting* to understand how numbers work."

The toddlers seemed to understand the smaller numbers when they used in a context that was meaningful. For example, Eleanor (29 months) used numbers when meaningful.

> Eleanor walks carefully across the room carrying a magnetic sphere on top of a cylinder. She almost bumps into Beatrice, who is holding the same shapes. Beatrice is pretending to lick the sphere. Eleanor smiles widely and exclaims, "I have one ice cream and Beatrice has one ice cream!"

Shapes. The third most observed category was shapes. All of the documented observations involved the children naming the shapes as they noticed them during the course of their play. Joshua (25 months) played with magnetic shapes at the light table. He handed a three-sided shape to the teacher and said, "It's a triangle, Tonya." Eleanor had a similar experience when she picked up a piece of her shape puzzle and stated, "It's a circle."

Both teachers discussed shapes as a part of mathematics but had differing opinions on the importance of shapes in the toddler classroom. Tonya considered shapes to be a "big thing" and devoted a lot of time to shape-related activities. She described an activity in which the toddlers went on a shape hunt.

> When we had shapes and colors (as a theme), they would go around the room and find something else that resembled the shape or color that we were looking for. They went and found rectangles, triangles, and squares around the room and we'd come back and talk about all the differences that we saw in the shapes they found.

Katie did not agree with her co-teacher's assessment. While she went along with Tonya's plans for doing a shape theme in the class, she confided, "A lot of parents are concerned with their children knowing their shapes. I don't necessarily think that's a big issue. They're going to eventually learn them. I don't think that's something that should be pressed."

Discussion

Most of the observed play and teacher discussion focused only on three of Lee's (2012) seven categories of math: classification, number sense, and shapes. A possible explanation is that teachers tend to be narrow in their understanding of appropriate early childhood math concepts. These categories are the same areas that are typically highlighted in early childhood math programs (Baratta-Lorton, 1995). Both of these teachers were familiar with preschool mathematics: one as the former lead teacher in a preschool class and one as an undergraduate in a university program that focused primarily on children ages preschool and older. An implication is that teacher must not only be *exposed to* but also *understand* a broad range of mathematical content so that all categories are encouraged during toddlers' mathematical play.

With such a limited focus on mathematics with toddlers, there is a plethora of opportunities for research. Possible areas worth further exploration include the mathematical experience of children in more traditional childcare centers; a focus on the mathematical understanding of younger toddlers, and an investigation of how toddlers explore mathematics in other environments such as with their families at home.

References

Baratta-Lorton, M. (1995). *Mathematics their way: An activity-centered mathematics program for early childhood education*. Reading, MA: Adison-Wesley.

Björklund, C. (2008). Toddler's opportunities to learn mathematics. *International Journal of Early Childhood, 40*(1), 81-95.

Clements, D. H. (2005). Math play: How young children approach math. *Early Childhood Today, 19*(4), 50-57.

Geist, E. (2001). Children are born mathematicians: Promoting the construction of early mathematical concepts in children under five. *Young Children, 54*(4), 12-19.

Geist, E. (2009). Infants and toddlers: Exploring mathematics. *Young Children, 64*(3), 39-42.

Greenberg, J. (2012). More, all gone, empty, full: Math talk with infants and toddlers-every day, in every way. *Young Children, 67*(3), 62-64.

Hansen, L. E. (2005). ABCs of early mathematical experiences. *Teaching Children Mathematics, 12*(4), 208-212.

Lee, S. (2012). Toddlers as mathematicians? *Australasian Journal of Early Childhood, 37*(1), 30-37.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Ozdogan, E. (2011). Play, mathematic and mathematical play in early childhood education. *Pocedia Social and Behavioral Sciences, 15*, 3118-3120.

Reikerås, E., Løge, I.K. & Knivsberg, A. (2012). The mathematical competencies of toddlers expressed in their play and daily life activities in Norweigian kindergartens.

Sarama, J. A., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young child.*

Jill Davis is an early childhood teacher currently working to complete her doctoral program in Early Childhood with an emphasiss in mathematics at the University of Oklahoma, Tulsa campus.

Professional Resources

Learning and Teaching Early Math: The Learning Trajectories Approach, Douglas H. Clements and Julie Sarama, 2009, 327 pp, $45.40 paperback, ISBN-978-0-415-99592-4. Routledge.

In their recent publication, Douglas Clements and Julie Sarama provide an excellent resource for teachers of early mathematics. *Learning and Teaching Early Math: The Learning Trajectories Approach* summarizes the research done on early mathematics learning and teaching, namely "what is known about how children learn, and how to build on what they know." The authors present learning trajectories that describe the developmental nature in how children learn mathematics starting from birth. Each learning trajectory is aligned with a specific mathematical topic, research that describes how children learn and understand the topic, and activities that encourage development of the topic along the trajectory. The book provides teachers with direction for understanding where their students' mathematical abilities might be in relation to their chronological age and instructions on how to help students progress in their mathematical development.

Chapter 1 provides an introduction to the Building Blocks project based on learning trajectories whose purpose was to find mathematics and develop mathematics from children's activity. A detailed definition of a learning trajectory is presented as having three elements: a mathematical goal, a developmental path to reach that goal, and a set of instructional activities. Chapters 2-12 take the reader through specific early mathematics topics including number sense, counting, early addition and subtraction, place value, spatial thinking and shapes, and geometric measurement. Each chapter is highly structured and organized in like fashion. After presenting a summary of the research on the teaching and learning of the chapter topic, education experiences are discussed –those that hinder a child's mathematical development and those that encourage it. This is also the part of the chapter where instructional activities are described—some in detail like those used in the Building Blocks project and others simply offered as suggestions. Finally, each chapter ends with a detailed description of the learning trajectory that outlines the developmental progression of the mathematical topic for that chapter. The last few chapters (13 – 16) are structured differently but still rely on research summary to discuss issues concerning how children think about mathematics and their mathematics affect. The authors also discuss equity concerns, contexts and curricula and specific instructional practices.

The intentions of the authors are clear—that the learning trajectory approach to teaching early mathematical topics is designed to help teachers, administrators and curriculum coordinators bolster early math achievement for all students. In particular, the authors purposed their book to be a highly valued and effective tool for early mathematics educators. The goal is achieved in many ways but there are some practical points that are worth mentioning. First, the book acts as a quick reference guide, a quality attributable to the way in which the chapters are structured and organized. Each chapter after the first applies the same flow of ideas to a new early math topic culminating with the learning trajectory. The learning trajectory is presented in a table format that also lends easily to quick reference. Any chapter can be taken separately but the authors caution the teacher-reader against reading only about the mathematical topic of interest because there are many elements that comprise the successful implementation of a learning trajectory and all elements must be fully considered. Therefore, in addition to the chapter of interest, the reader should also read the final three chapters for a more complete picture of the equity concerns and instructional practices that go along with using the learning trajectory approach. Another pragmatic quality of the book is the attention the authors give to summarizing the pertinent research regarding teaching and learning of early mathematics. The authors understand that the research field on these topics is vast and valuable, and they use practical language to put that research into the hands of those who can use it most effectively.

Although Clements and Sarama have created a seemingly thorough resource, the inclusion of some minor items might make it even *more* useful to teachers. Since the introduction of the Common Core Standards for Mathematics (CCSM), teachers are searching for ways to instruct their students according to the new guidelines. An additional section or chapter in the learning trajectories book that matches each learning trajectory with the appropriate Standards would further insure that teachers will successfully make the connections to practical implementation of the developmental progressions their students need and must experience. With direct reference to the CCSM, the guesswork is removed and a clear picture of *what* topics to teach and *when* to teach them is realized. Teachers would likely feel more confident in using the learning trajectory approach knowing those direct links to the CCSM (upon which their job performance is based) are presented alongside the research and instructional activities. Additionally, in regard to the instructional activities, it would be particularly useful to have those listed in a table format for quick and easy recall. Although the authors admit that some learning trajectories rely heavily on activities from the Building Blocks project software, they do provide alternative "practical suggestions" for activities that serve the same goal of developmental progression. These suggestions are provided within the context of the chapters so it is necessary to fully read the chapter to glean ideas. Having a table format in which to summarize or present these alternative activities would save valuable time especially for those teachers who are committed to implementing the learning trajectories to meet the CCSM. These educators are the individuals that have already read the book and have agreed with the research—namely that "effective mathematics teaching involves meeting the students where they are and helping them build on what they know."

Jenny Peters, Mathematics Instructor
Rogers State University

Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K–5, Sherry Parrish, 2010, 390 pp , $73.95, paperback with DVD, ISBN 978-1-935099-16-1. Math Solutions.

Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K–5 , by Sherry Parish, is a practical guide for using focused classroom discussions to identify student strategies and build on those strategies to progress thinking, understanding, and reasoning. Number talks are becoming a vital piece of the mathematics classroom and build number sense, problem solving strategies, and computational fluency. Students build flexibility in thinking by sharing and building mathematical strategies. The book begins at the earliest stages of number talks, guiding teachers and facilitators in the implementation of number talks in their classroom. Parrish provides example strategies and purposefully constructed problems that have students developing specific strategies. The accompanying DVD allows teachers to view sample talks and how the strategies are built.

Number Talks is packed full of resources for teachers and facilitators. The resources are categorized by strategy and grade level, as well as NCTM alignment, for easy reference by the user. Section one of the book gives users a practical overview of exactly what a number talk is and how to prepare for them. These strategies are essential management routines and expectations for a number talk and building a mathematical community of learners. Facilitators are challenged to set expectations, goals, and roles for their number talks.

Thinking strategies for grades K-2 are the focus of the next section wherein Parrish identifies the strategies that are common to building number sense and aiding in computational fluency. Aligned with these strategy examples are the DVD example lessons enabling facilitators to see the strategies in action. The section outlines how students discuss and build efficient strategies through the discussions and modeling. Sample problems are given for each grade level and the strategies that the students should be moving into.

Grades 3 - 5 are the focus of the next section. The students continue to build more efficient strategies for addition and subtraction while also developing strategies for multiplication and division. In this section, the author again provides common strategies and encourages facilitators to be purposeful about anticipating student thinking and possible misconceptions. Section 3 also addresses the importance that real life contexts have on understanding. Parrish provides a plethora of problems for teachers to use in number talks. They are organized by grade and strategy in an effort to progress student thinking towards strategies that are more efficient.

The final section of the book is the Facilitator's Guide. This guide zooms in on each grade level, discussing what a typical math class would look like including how much time should be spent on number talks, whole group, practice, etc. This section is designed to get the facilitators to think about their practices and continue to improve their teaching. The appendix offers more resources for facilitators including observations, answers to common questions and problems that they face.

This resource is easy to read, explicit in its content and focused on teacher usability. Teachers looking to begin using number talks to improve student computation and reasoning will find this resource straight forward and user friendly. As teachers continue to grow in their use of number talks, this resource can guide them by answering questions about problems they encounter, managing their time, as well as progressing student thinking. Teachers who are experienced in number talks can benefit from the strategy alignment and discussion questions that are included to improve the effectiveness of their math number talk time. This resource has a lot to offer anyone looking to build students' mathematical learning. Students are challenged to build community, explore their thinking, and build their understanding through systematic development of strategies.

DeAnne Finley
Union Public Schools

Classroom Discussions in Math; A Teachers Guide to Using Talk Moves to Support the Common Core and More. Suzanne Chapin, Catherine O'Conner, Nancy Canavan Anderson, 2013, 391 pp, $71.95 paperback with video, ISBN 978-1-935099-56-7. Math Solutions.

In *Classroom Discussions in Math: A Teachers Guide to Using Talk Moves to Support the Common Core and More*, authors Suzanne Chapin, Catherine O'Conner and Nancy Canavan Anderson outline the necessity and purpose of meaningful talks in the mathematics classroom. This book guides educators through the process of creating and facilitating meaningful discourse in the classroom that is engaging to diverse learners. Authors discuss the benefits of math talks and the importance of having mathematical conversations in a setting that is conducive to mathematical thinking and accessible to all learners.

Section I of the book provides an overview of how to launch Math Talks in the classroom utilizing both whole group and small group discussions. Through this section teachers learn how talk can reveal student understandings as well as misconceptions in their mathematical thinking. They learn how to set norms for productive conversation which will support robust learning. Talk Moves are introduced to teachers as tools to invite participation and increase productivity of the talk in the classroom. Through these discussions the authors state that students will be able to engage and reason through the thinking of others while deepening their own understanding of the topics being discussed.

Knowing what math to talk about is a vital skill in developing math talks for the classroom. Section II gives guidance on how teachers can focus their talks on specific math concepts, procedures and strategies. Through the math talks in these chapters teachers help students build connections among ideas, discover computational strategies and their connections to algorithms. Most importantly, these talks help students connect computational procedures to concepts. Chapter 5 focuses on helping students discover methods for problem solving and determining appropriate and efficient strategies based on context. In this chapter teachers encourage students to embed the Common Core Mathematical Practices into their problem solving approaches.

In order to support teachers in their endeavor to create meaningful classroom conversations Section III outlines how teachers can foster student buy in, create talk based lesson plans and also addresses some frequently asked questions. The authors explain that students must see the importance of talk and its connection to their future as related to their careers and collaboration in problem solving outside of the math classroom. In the section about talk based lesson plans there are detailed directions, as well as considerations that teachers must make when planning mathematical talks in the classroom. Finally, the authors address some of the issues that are likely to arise as the classroom discourse unfolds. These issues include getting students to talk, diverse classroom populations and the need for differentiation, as well as managing classroom behavior during math talks and many others.

A great addition to the book is the DVD video resource that is included. The clips include excerpts from lessons in real classrooms across grade levels K-6 from schools in Massachusetts. Lessons are aligned with Common Core State standards and correlations are listed in the introduction to the book. Each video clip includes questions for discussion and reflection. These questions could be used during personal reflection as well as collaboration with colleagues. Additionally, the authors have included 2 lesson plans for each grade K-6. These lessons are designed around the concepts and procedures outlined throughout the book and are tied to Common Core Standards as well. These lessons can be used as they are written or can be easily modified to fit the needs of any classroom.

By using the strategies outlined in Classroom Discussions in Math, teachers will learn how to build an atmosphere of mutual respect and support where students will feel comfortable and confident in sharing their mathematical thinking. Building on the importance of social interaction, these talks provide students with opportunities to use each other as resources, share their ideas with others and participate in the joint construction of knowledge. Students become actively involved in the learning process through equitable participation and teachers serve as guides to lead their students in mathematically sound directions.

Moriah Widener-Jenks Public Schools
Melynee Naegele-Claremore Public Schools

Manuscript Reviewers for the Oklahoma Journal of School Mathematics

The educators listed below give their time and expertise to read, review, and write reports on articles submitted to the *Oklahoma Journal of School Mathematics*. Their efforts are greatly appreciated by the editors and the Oklahoma Council of Teachers of Mathematics.

Summer Bateiha
Darlinda Cassel
Kansas Conrady
Julia Cook
Kristi Dickey
Annetta Hackler
Linda Hall
Adele Hanlon
Mary Harper
Larry Hessler
Wendy James
Courtney Lockridge
Carol Lucas
Levi Patrick
Gina Morris
Sacra Nicholas
Melisa Rice
Jan Sands
Adrienne Sanogo
Nicole Shobert
Betsy Showalter
Laura Smith
Karen Strande
Mike Turegun
Vicki Vaughan
Stacy Weinand
Jayne Ann Williamson
John Woods

Volunteer to be a reviewer by completing this form and returning it to:
Juliana Utley (juliana.utley@okstate.edu) or Stacy Reeder (reeder@ou.edu).

Name:
Affiliation:
Address:
Phone:
Fax:
E-mail:

Please select the grade band(s) that best aligns with your expertise.

- PreK – K ☐
- Grades 1-3 ☐
- Grades 4-5 ☐
- Grades 6-8 ☐
- Grades 9-12 ☐
- University ☐

Please include any other information regarding your expertise you think might be pertinent (i.e., technology, integration, ect.).

Oklahoma Council of Teachers of Mathematics

Please complete this form and mail with your **check payable to OCTM** to:

Vicki Vaughan
7620 Northway Terrace
Oklahoma City, OK 73162

Name: ____

Home Address: ____

City, State, Zip: ____

Home Phone: ____

School: ____

School Address: ____

School City, State, Zip: ____

School Phone: ____

E-mail Address(es): ____

Choose the category that best describes you (please circle):

Elementary	Middle School	High School	OSSM
Community College	OSU or OU	Retired	Student
Private College/Univ.	Government	State College/University	

Other (please specify):______________________

Membership Fee:

Full-time mathematics: $20.00 for 1 year membership = _____________

$35.00 for 2 year membership = _____________

$50.00 for 3 year membership = _____________

Less than ½ time mathematics, Student, Retired, non-teaching:

$10.00 for 1 year membership = _____________

OkJSM Publication Guidelines

Submission of Manuscript
Email manuscript to the Journal Editors
Juliana Utley (juliana.utley@okstate.edu) and/or Stacy Reeder (reeder@ou.edu).

Guidelines for Authors
Authors are requested to submit only unpublished articles not under review by any other publication. Manuscript should be typed, double spaced, not right justified, not hyphenated, and should follow APA, 6th edition guidelines (Publication Manual of the American Psychological Association) using Microsoft Word. Tables and graphs should be used only when absolutely necessary. Include a cover page giving the article title, professional affiliation, complete address, e-mail, and phone number of the author(s). Each type of submission has specific requirements which are described below. The editors reserve the right to edit all copy.

Review Information
All manuscripts will be sent out for blind review to at least 2 external reviewers and one in-house reviewer. Manuscripts will be returned to authors with a summary of reviewers comments.

Teacher to Teacher: Submit descriptions of teaching activities that have helped students learn an essential mathematics skill, concept, strategy, or attitude. Submissions should be no longer than 1500 words, typed and double-spaced, and follow the following format:

- Title (if adapting from another source, cite reference and provide a bibliography).
- Purpose of activity, including the mathematics skill, concept, or strategy students will learn.
- Description of activity with examples, questions, responses. Please provide enough detail so someone else can implement the activity.
- How activity was evaluated to know if purpose was achieved.

Teacher Research: Submit manuscripts that describe research or inquiry conducted in classrooms. Submissions should be 1000-2000 words, typed and double-spaced following guidelines of the APA, 6th Edition, and follow this format:

- Description of the question or issue guiding the research/inquiry, including a short review of pertinent literature.
- Description of who participated in the study, what the sources of data were, how the data were gathered and examined.
- Description of the findings and conclusions from the research/inquiry.

Research Summary: Submit manuscripts that summarize either one current published piece of research or two to three related studies. Submissions should be 1000-1500 words, typed and double-spaced following guidelines of the APA, 6th Edition, and following this format:

- Introduce and describe the study or studies, including purpose, information about who participated and in the study, how and what type of data was gathered, and the findings or conclusions.
- Discuss the implications of the study or studies for classroom teachers. The implications could include a discussion of what the study told us about mathematics learners and mathematics learning and/or what the study implies teachers should do to support learning.

Professional Resources: Submit reviews of professional resources of interest to teachers or mathematics curriculum specialists. Resources reviewed could include books for teachers, books for children, curriculum packages, computer programs or other technology, or games for children. Submissions should be 500-1000 words, typed and double-spaced following guidelines of the APA, 5th Edition, and following this format:

- Title, author, publisher of the resource, year published; number of pages, cost.
- Short description of the resource.
- Critical review of the resource, including strengths and weaknesses.
- Short discussion of how the resource might be useful to a teacher.